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A statistical index of growth condition in an aquaculture experiment

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The ratio of the standard deviation of the logarithm of weight to the standard deviation of the logarithm of length is proposed as a simple statistical index of growth condition of the fish or related species in an aquaculture experiment.

This paper will present a simple statistical index for evaluating the condition of growth in an aquaculture experiment and indicating the extent of effect of any plausible rival hypothesis. The index is easy to compute and can be statistically tested.

The Growth Condition Index

Suppose that the growth in weight (W) and length (L) of the fish can be described by the allometric growth model

$$W = \alpha L^{\beta}, \tag{1}$$

where α and β are unknown parameters. In terms of the logarithms of W and L, model (1) can be written as a simple linear model of the form

$$Y = \log \alpha + \beta X, \tag{2}$$

where Y = log W and X = log L. Most studies on weight-length relationships involve fitting model (2) to the data by the method of ordinary least squares.

If during the growth process the individual fish maintains its shape or specific gravity, then the relative dispersions of the logarithms of weight and length of the population will remain constant. Thus, a simple index that may be employed as an indicator of growth condition in an aquaculture experiment is

$$\delta = \frac{\sigma_Y}{\sigma_X} \tag{3}$$

where σ_Y and σ_X are the standard deviations of Y = log W and X = log L.

It can be shown that the growth condition index is identical to the geometric mean of the bivariate normal regression of Y on X and the reciprocal of the regression of X on Y. Ricker (1973) refers to δ as the geometric mean (GM) functional regression.

Interpretation and Applications of δ

On the assumption that X and Y have a joint normal distribution, then o would have the same interpretation as β in model (1). A value of $\delta = 3$ indicates *isometric* growth, i.e., the weight of the fish increases as the cube of length. Values of $\delta \doteq 3$ indicate allometric growth. When

$\delta > 3$, the fish are heavier for its length as would be observed in a population with adequate food supply but growth in length is retarded due to lack of space. On the other hand, in a population where the larger individuals lack a suitable food supply, a value of $\delta < 3$ may be observed (Ricker, 1971, 1979).

An important application of δ in an aquaculture experiment is for monitoring the conditions of growth in the experimental units. If the fish were uniformly selected at the start of the experiment, then the values of δ will be nearly constant for the different experimental units. However, during the experiment the values of δ will vary because of developmental growth or major environmental changes in the experimental units. Thus, if the growth conditions in experimental units applied the same treatment differ widely, a large experimental error may result.

Another application of the growth condition index δ is in the computation of the condition factor, which measures the general 'well-being' of the individual fish in the different experimental units or populations. It can be used in lieu of β in the allometric condition factor formula, i.e.,

$$K = \frac{W}{L^{\delta}} \tag{4}$$

The condition factor (4) has been found most effective "in comparing two or more monospecific populations living under apparently similar or different conditions of food, density, climate, etc., in determining the timing and duration of gonad maturation in populations; and, in following the gradual build up or decline of feeding activity over an extended period, or population changes possibly attributable to alterations in the food supply, as reflected in the gross nutritional balance of the fish (Weatherley, 1972).

The growth condition index δ is easier to compute than β since it involves only the standard deviations of the logarithms of weight and length measurements. Furthermore, β is commonly estimated by the method of ordinary least squares from (2), assuming that $X = \log L$ is fixed and measured without error. But, both $Y = \log W$ and $X = \log L$ are random and there is really no reason to consider either one as independent or dependent variable.

Maximum Likelihood Estimator of δ

Let $(X_1, Y_1), (X_2, Y_2). \ldots . (X_n, Y_n)$ be a random sample from the bivariate normal distribution. Then, the maximum likelihood estimator of δ is

$$\hat{\delta} = \frac{S_y}{S_x} \tag{5}$$

where $S_y = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (Y_i - \bar{Y})^2}$ and $S_x = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^2}$.

Testing the Hypothesis $H_o : \delta = \delta_o$

The test of the null hypothesis $H_o : \delta = \delta_o$ is derived using an ingenious approach originally due to Pitman (1939) and discussed in Snedecor and Cochran (1967).

This null hypothesis $H_o : \delta = \delta_o$ versus the alternative hypothesis $H_a : \delta \neq \delta_o$ can be tested using

$$t = \frac{(\hat{\delta}^2 - \delta_o^2)}{2\hat{\delta}\delta_o} \sqrt{\frac{n-2}{1-\hat{\rho}^2}} \tag{6}$$

Where $\hat{\rho} = \dfrac{\Sigma (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\Sigma (x_i - \bar{x})^2 \Sigma (y_i - \bar{y})^2}}$

The decision rule would be to reject Ho and accept H_a if $|t| > |t_{\alpha}(n\text{-}2)$, where $t_{\alpha}(n\text{-}2)$ is the two-tailed significance point of the t-statistic with n-2 degrees of freedom.

Confidence Interval for δ

A 100 (1-α)% confidence interval for the unknown parameter δ can be obtained from (7) and is given by

$$\hat{\delta}\sqrt{c - \sqrt{c^2 - 1}} < \delta < \hat{\delta}\sqrt{c + \sqrt{c^2 - 1}}$$

where $c = 1 + \dfrac{2(1 - \hat{\rho}^2)\, t_{\alpha}^2}{(h - 2)}$

The following three examples will serve to illustrate the computations and applications of the growth condition index.

(a) One hundred and fifty (150) milkfish fingerlings were sampled from a large batch prior to stocking in experimental cages. The computed values of S_y, S_x, and $\hat{\rho}$ are $s_y = 0.5887$, $s_x = 0.2033$, and r = 0.9814, respectively, where Y is the logarithm of weight in grams and X is the logarithm of total length in millimeters. Test for isometric growth, i.e., $\delta = 3$, is as follows:

$H_o: \delta = 3$

$H_a: \delta \neq 3$

α a = 0.05

CR: $|t| > 1.96$ df = 148

$$\hat{\delta} = \frac{0.5887}{0.2033}$$

$$= 2.8957$$

$$t = \frac{(2.8957^2 - 3^2)}{2(2.8957\ (3)} \sqrt{\frac{148}{1 - 0.9811}\ 2}$$

$$= -2.23$$

Conclusion: Reject H_o and conclude that the growth is allometric although the value of the growth condition index is nearly 3 because the sample size is large and the correlation between Y and X is very high.

The computed value of the constant c in (7) is

$$c = 1 + \frac{2(1 - 0.9811^2)\ (1.96)^2}{148}$$

$$= 1.0019$$

Hence, a 95% confidence interval for δ is

$$2.81 < \delta < 2.99$$

(b) In the following example, the growth condition index δ is employed to compare male and female broodstock prawns in two different ponds. Here Y is the logarithm of weight in grams and X is the logarithm of carapace length in millimeters (De Mesa, 1980). Table 1 summarizes the data and tests of significance.

Table 1

Sex	Pond	n	s_y	s_x	r	$\hat{\delta}$	t-values for testing $H_0{:}\delta = 3$	$H_0 : \delta = 2.5$
Male	1	75	0.1760	0.0625	0.9231	2.8160	–1.41	2.65**
	2	103	0.1409	0.0500	0.8971	2.8180	–1.42	2.73**
	1	178	0.1364	0.0546	0.9184	2.4982	–6.17**	–0.02
Female	2	104	0.1439	0.0544	0.9356	2.6452	–3.61**	1.62

**Highly significant ($\alpha = 0.01$)

From the above results, it can be concluded that there is no evidence to reject the null hypothesis that the growth of the male broodstock prawns in the two ponds is isometric. On the other hand, it is evident that the growth of the female prawns in the two ponds is allometric.

Since the t-values between ponds for male are nearly the same, a combined estimate of δ can be computed as follows:

$$\hat{\delta} = \sqrt{\frac{(75-1)\ (0.1760)^2 + (103-1)\ (0.1409)^2}{(75-1)\ (0.0625)^2 + (103-1)\ (0.0500)^2}}$$

$$= \quad 2.82$$

The growth conditions of female prawns in the two ponds are different as indicated by $\hat{\delta}$ and their associated t-values.

(c) Presented in Table 2 below are the estimates of the growth condition index δ in a completely randomized design experiment to study the effect of stocking density on the growth and survival of *Tilapia nilotica* fingerlings reared in cages in Laguna Lake. Ten (10) fingerlings were sampled twice a month from each of the 16 cages for weight (gm) and total length (mm) measurements (Basiao, 1980).

Table 2

Stocking Density	Replicates	Sampling Period 1st	2nd	3rd
(No./m^2				
	1	3.0	3.1	3.3
50	2	2.7	3.2	3.1
	3	3.0	2.9	2.6
	4	3.3	3.3	2.4
	1	3.1	2.9	3.3
100	2	2.6	3.3	3.3
	3	2.6	2.7	3.0
	4	2.4	2.8	3.4
	1	3.9	3.0	2.8
150	2	2.6	3.0	2.8
	3	2.8	3.1	2.8
	4	2.4	3.0	2.9
	1	3.0	3.1	3.1
200	2	2.8	2.6	3.2
	3	2.9	3.0	3.2
	4	2.7	3.5	2.9

The means of the estimated growth condition index are summarized in Table 3.

Table 3

Stocking Density	Sampling Period 1st	2nd	3rd	Over-all mean
50	3.0	3.1	2.9	3.0
100	2.7	2.9	3.3	3.0
150	2.9	3.0	2.8	2.9
200	2.9	3.1	3.1	3.0
Over-all mean	2.9	3.0	3.0	3.0

The data in Table 2 are analyzed using a two-way mixed model analysis of variance (Morrison, 1976) and the results are presented in Table 4 below.

Table 4

Source of Variation	df	Sum of squares	Mean square	F
Sampling periods	2	0.26542	0.13271	1.33
Stocking densities	3	0.04500	0.01500	0.21
Replicates within densities	12	0.84834	0.07069	
Periods x densities	6	0.77125	0.12854	1.29
Replicates x periods within densities	24	2.39666	0.09986	
Total	47	4.32667		

All the F-tests are not significant. This indicates that there is no evidence to conclude that growth conditions in the cages are not the same for all stocking densities and sampling periods. Also, the null hypothesis of no interaction between stocking densities and sampling periods cannot be rejected. It should be noted, however, that the above analysis of variance is approximate and does not take into account the distributional properties of the growth condition index. Furthermore, the analysis is strictly valid only if the estimates within a replicate have a common variance and the same correlation for all pairs.

The growth condition index is easy to compute and can be tested statistically. When the logarithms of weight and length measurements of the fish have a bivariate normal distribution, the index is identical to the geometric mean functional regression. Hence, it has the same interpretation and applications in aquaculture research as the slope parameter of the allometric growth model.